Mr. Michael T. Davis Calculus	Section 2.4 & 3.1 Practice Quiz December 7, 2017

Name: Solutions/Answers

1. Find the average rate of change of $p(x) = 3x^2 - x$ over the interval over the interval $[-1,3]$

$AROC = \frac{p(3) - p(-1)}{3 - -1}$ slope of the secant line

$AROC = \frac{3(3)^2 - 3 - (3(-1)^2 - (-1))}{4}$

$AROC = \frac{24 - (4)}{4}$

$AROC = \frac{20}{4}$

$AROC = 5$

2. Find $f'(x)$ for the function $f(x) = -3x + 10$

$f'(x) = -3$

3. Find $\frac{dy}{dx}$ for the function $y = 7$

$\frac{dy}{dx} = 0$

4. Find y' for the function $y = x^2$

$y' = 2x$

5. Evaluate $\frac{d(8x)}{dx}$

$\frac{d(8x)}{dx} = 8$

6. Write an equation of the line tangent to the curve of $y=f(x)$ at the point $x=4$ given $f(4)=-5$ and $f'(4)=\frac{1}{2}$

$$y - {}^{-}5 = \tfrac{1}{2}(x-4)$$

$$y+5 = \tfrac{1}{2}(x-4)$$

7. Use the definition of derivative $g'(a)=\lim_{x\to a}\frac{g(x)-g(a)}{x-a}$ to derive the derivative function, $g'(a)$, of the function $g(x)=3x^2+5$. Then evaluate $g'(1)$.

$$g'(a) = \lim_{x\to a} \frac{3x^2+5-(3a^2+5)}{x-a}$$

$$= \lim_{x\to a} \frac{3x^2+5-3a^2-5}{x-a}$$

$$= \lim_{x\to a} \frac{3x^2-3a^2}{x-a}$$

$$= \lim_{x\to a} \frac{3(x-a)(x+a)}{x-a}$$

$$= \lim_{x\to a} 3(x+a)$$

$$= 3(a+a)$$

$$= 6a$$

$g'(a) = 6a$

$g'(1) = 6$

8. Given #7 above, determine an equation of the line tangent to the curve of $g(x)=3x^2+5$ at $x=1$.

$g(1) = 3(1)^2+5 = 8 \quad (x,y) = (1,8)$

$y-8 = 6(x-1)$

9. Use the definition of derivative $p'(x)=\lim_{h\to 0}\frac{p(x+h)-p(x)}{h}$ to derive the derivative function, $p'(x)$, of the function $p(x)=4x^2-x$. Then evaluate $p'\left(\frac{1}{2}\right)$

$$p'(x) = \lim_{h\to 0}\frac{4(x+h)^2-(x+h)-(4x^2-x)}{h}$$

$$= \lim_{h\to 0}\frac{\cancel{4x^2}+8xh+h^2\cancel{-x}-h-\cancel{4x^2}\cancel{+x}}{h}$$

$$= \lim_{h\to 0}\frac{8xh+h^2-h}{h}$$

$$= \lim_{h\to 0}(8x+h-1)$$

$$= 8x-1$$

$$p'(x)=8x-1 \qquad p'\left(\tfrac{1}{2}\right)=8\left(\tfrac{1}{2}\right)-1=4-1=3$$

10. Given #9 above, determine an equation of the line normal to the curve of $p(x)=4x^2-x$ at $x=\frac{1}{2}$.

$$p\left(\tfrac{1}{2}\right)=4\left(\tfrac{1}{2}\right)^2-\tfrac{1}{2}=4\left(\tfrac{1}{4}\right)-\tfrac{1}{2}=1-\tfrac{1}{2}=\tfrac{1}{2}$$

$$(x,y)=\left(\tfrac{1}{2},\tfrac{1}{2}\right)$$

$m=\frac{-1}{3}$ Normal $y-\frac{1}{2}=\frac{-1}{3}\left(x-\frac{1}{2}\right)$

11. Use the definition of derivative $k'(a)=\lim_{x\to a}\frac{k(x)-k(a)}{x-a}$ to derive the derivative function, $k'(a)$, of the function $k(x)=\sqrt{x}$. Then evaluate $k'(9)$

$$k'(a) = \lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}}{x-a}$$

$$= \lim_{x\to a} \frac{(\sqrt{x}-\sqrt{a})}{(x-a)} \frac{(\sqrt{x}+\sqrt{a})}{(\sqrt{x}+\sqrt{a})}$$

$$= \lim_{x\to a} \frac{x-a}{(x-a)(\sqrt{x}+\sqrt{a})}$$

$$= \lim_{x\to a} \frac{1}{\sqrt{x}+\sqrt{a}}$$

$$= \frac{1}{\sqrt{a}+\sqrt{a}}$$

$$= \frac{1}{2\sqrt{a}}$$

$$k'(a) = \frac{1}{2\sqrt{a}}$$

$$k'(x) = \frac{1}{2\sqrt{x}}$$

$$k'(9) = \frac{1}{6}$$

12. Use the definition of derivative $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ to derive the derivative function, $f'(x)$, of the function $f(x)=\frac{1}{x}$. Then evaluate $f'(-1)$

$$f'(x) = \lim_{h\to 0} \frac{\frac{1}{x+h}-\frac{1}{x}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1\cdot x}{(x+h)\cdot x} - \frac{1(x+h)}{x(x+h)}}{h}$$

$$= \lim_{h\to 0} \frac{x-x-h}{x(x+h)}\cdot\frac{1}{h}$$

$$= \lim_{h\to 0} \frac{-h}{x(x+h)}\cdot\frac{1}{h}$$

$$= \lim_{h\to 0} \frac{-1}{x(x+h)}$$

$$= \frac{-1}{x(x+0)}$$

$$= \frac{-1}{x^2}$$

$$f'(x) = \frac{-1}{x^2}$$

$$f'(-1) = -\frac{1}{(-1)^2}$$

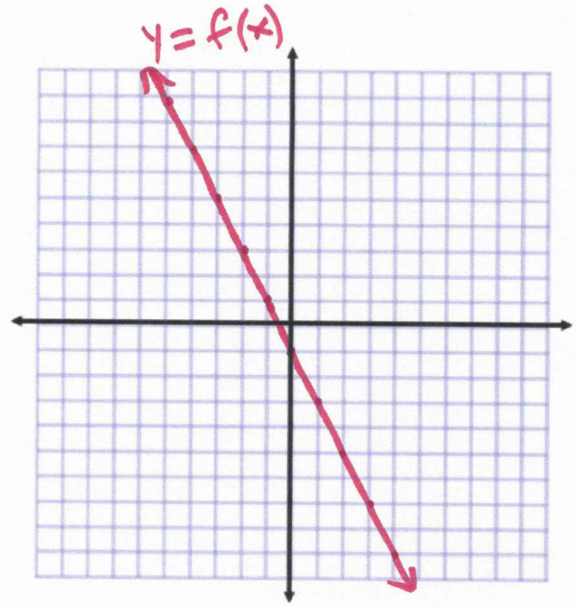
$$f'(-1) = -\frac{1}{1}$$

$$f'(-1) = -1$$

13. Graph of a continuous function $y=f(x)$ that has the following properties:

i. $f(-1)=1$,

ii. $f'(x)=-2$.



14. Write an equation for a linear function $y=f(x)$ that has the following properties:

i. $f(3)=-1$,

ii. $f'(x)=4$.

$y - {}^-1 = 4(x-3)$

$y+1 = 4(x-3)$

15. If $f(-3)=4$ and $f'(-3)=\dfrac{-4}{3}$, write an equation of the line tangent to the graph of $y=f(x)$ at the point where $x=-3$.

$y-4 = -\frac{4}{3}(x - {}^-3)$

$y-4 = -\frac{4}{3}(x+3)$

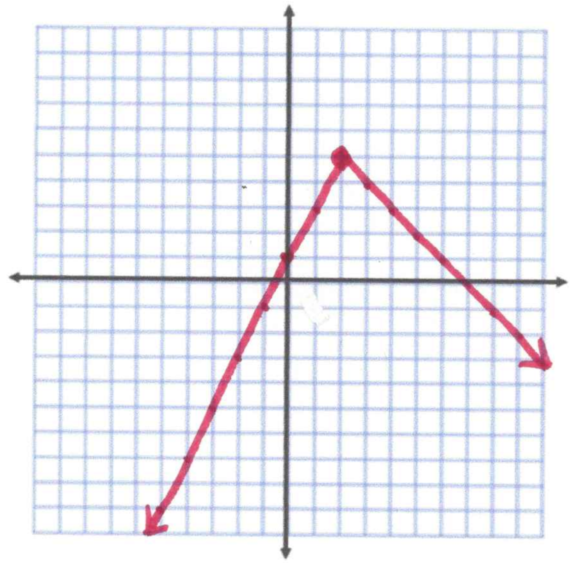
16. If $f(1)=-1$ and $f'(1)=\dfrac{1}{5}$, write an equation of the normal line to the graph of $y=f(x)$ at the point where $x=1$.

$y - {}^-1 = -5(x-1)$

$y+1 = -5(x-1)$

17. Sketch the graph of a continuous function $y = f(x)$ with $f(0) = 1$ and

$$f'(x) = \begin{cases} 2 \text{ if } x < 2 \\ -1 \text{ if } x > 2 \end{cases}$$



18. Determine if the piecewise function $f(x) = \begin{cases} x^2 + x \text{ if } x \le 1 \\ 3x - 1 \text{ if } x > 1 \end{cases}$ has a derivative at $x = 1$.

Justify your answer.

For $x \le 1$ $f'(x) = \lim_{h \to 0} \dfrac{(x+h)^2 + x + h - (x^2 + x)}{h}$

$f'(x) = \lim_{h \to 0} \dfrac{\cancel{x^2} + 2xh + h^2 + \cancel{x} + h - \cancel{x^2} - \cancel{x}}{h}$

$f'(x) = \lim_{h \to 0} (2x + h + 1)$

$f'(x) = 2x + 1$

$f'(1) = 2(1) + 1 = 3$

For $x > 1$

$f'(x) = 3$

since the right sided derivative is equal to the left sided derivative and $f(x)$ is continuous at $x = 1$, the function does have a derivative at $x = 1$

left side

$(1)^2 + 1 = 2$

Right side

$3(1) - 1 = 2$

therefore the function is continuous at $x = 1$

19. Determine if the piecewise function $f(x) = \begin{cases} x^2 - 3 \text{ if } x < 2 \\ 3x - 5 \text{ if } x \ge 2 \end{cases}$ has a derivative at $x = 2$.

Justify your answer.

left side at $x = 2$ $(2)^2 - 3 = 4 - 3 = 1$

Right side at $x = 2$ $3(2) - 5 = 6 - 5 = 1$

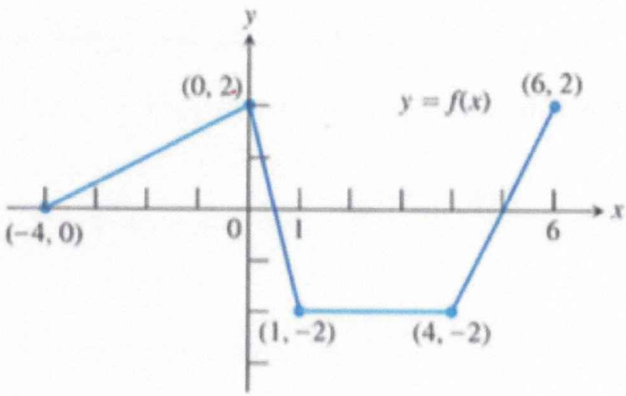
therefore the function is continuous at $x = 1$

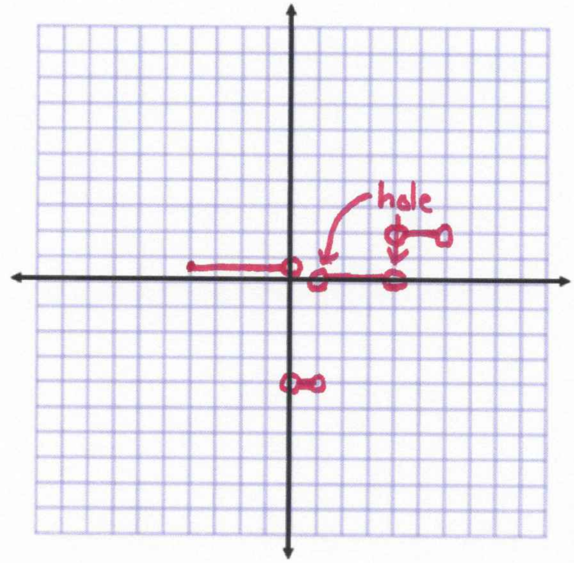
left sided derivative is $2x$ so at $x = 2$ $2(2) = 4$

right sided derivative is 3

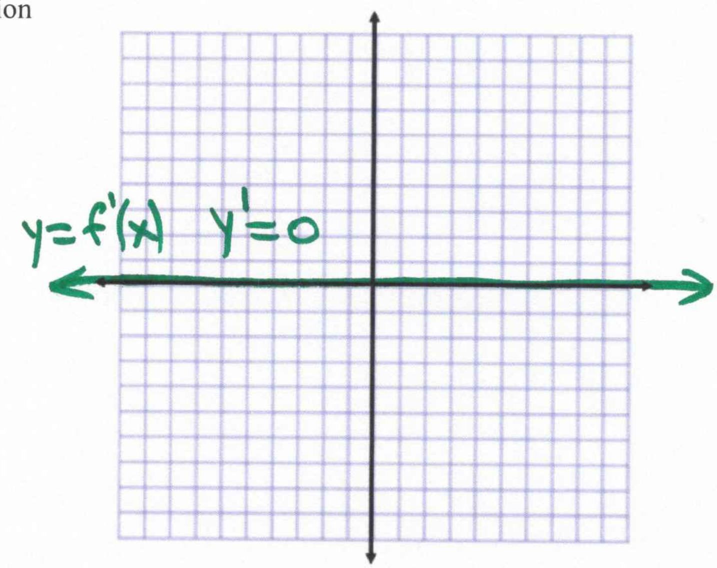
since the left sided derivative is not equal to the right sided derivative, the function does not have a derivative at $x = 2$.

20. The graph of the function $y = f(x)$ shown here is made of the line segments joined end to end. Graph the function's derivative function.

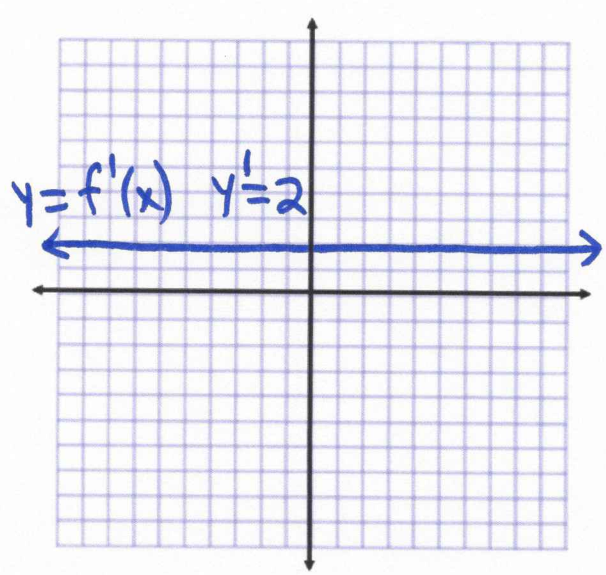




21. Graph the derivative function of the function
$f(x) = -4$



22. Graph the derivative function of the function
$f(x) = 2x - 9$



23. Graph the derivative function of the function $f(x) = x^2$

$f'(x) = 2x$